

Current Distributions on Transmitting and Receiving Antennas*

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Summary—A method for the direct measurement of the amplitude and phase of current and charge distribution along cylindrical antennas is presented for both transmission and reception. This method is applicable not only to the cylindrical antenna treated here but to many other antennas which may be used over a ground screen. The measured distribution on the transmitting antenna is continued into the coaxial line so that a complete picture of the system as a whole is obtained. The measured distribution on the receiving antenna is then shown to vary as a function of the terminating load as predicted by theory.

I. INTRODUCTION

UP TO THE PRESENT time, measurements of distributions on antennas have been limited to the absolute value of current or charge.^{1,2} Adopting the concept of the traveling-wave picture, a current wave can be assumed traveling to the end of the antenna and then being reflected back. The result obtained by assuming a 180-degree phase change at a minimum point indicates that there is no radiation. Actually, however, phase reversal takes place by means of phase rotation rather than by the amplitude going to zero, so that the current at any point on the antenna may be decomposed into two components, one in phase with the driving voltage, the other in phase quadrature.

The most recent experiment reported in this field involves the determination of phase by a graphical procedure based on the equation of continuity.³ In discussing the method, the author states that since the equation of continuity is a differential relation, graphical procedure must be followed and a high degree of accuracy cannot therefore be expected in the estimate of phase. In this paper, both amplitude and phase of current and charge on the antenna are measured directly and the measurements are continued into the coaxial transmission line so that a complete picture of the system is obtained. Measurements made on transmitting antennas indicate a terminal-zone effect identical with the discontinuities present in waveguide theory, where higher-order modes are introduced to satisfy the boundary condition at the discontinuity. Only for the case

of the unloaded receiving antenna is this discontinuity absent so that a much better correlation with theory is obtained. The measured results on the receiving antenna show the large variation possible in current distribution as a function of the load impedance.

II. MEASURING SETUP AND TECHNIQUE

It has been customary in the past to measure transmission-line impedance by using a small probe inserted in the coaxial line through a longitudinal slot on the outer conductor. In the setup to be described the alternative procedure is followed, incorporating a slotted inner conductor from which the probe protrudes. In this arrangement, the inner conductor may be extended over a ground screen and the extension used as an antenna, thereby permitting the probe to be moved both inside the line and on the antenna to measure the complete distribution of current or charge. The schematic diagram of Fig. 1 shows the rack-and-pinion arrangement for obtaining the continuously variable length of an-

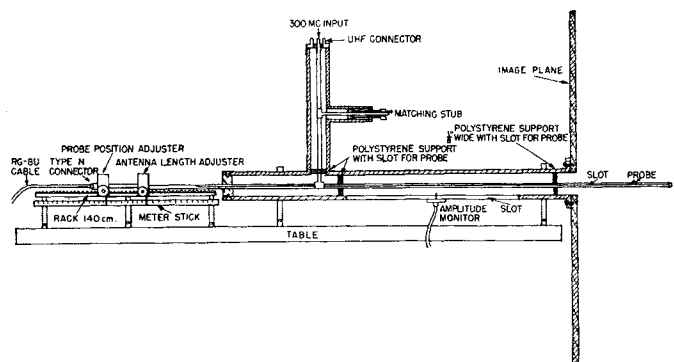


Fig. 1—Schematic diagram of measuring line.

tenna. The line leading from the probe to the detector is placed inside the inner conductor so that it is entirely shielded from radiation, thereby eliminating any perturbation of the radio-frequency field by this cable.

A. Amplitude Measurement

By the application of the first of two boundary conditions imposed on the metallic surface, the tangential component of the magnetic field is directly proportional to the surface-current density. Thus, by the use of a small shielded loop probe—in this case extending 2 mm over the surface of the antenna and 5 mm in width—the current distribution measurement can be made. The shielded loop is used to insure the elimination of the “dipole mode” pickup and to obtain a simple transformation of the loop to the coaxial detecting system.

The second boundary condition on the metallic surface requires that the charge density be proportional to

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¹ H. E. Gihring and G. H. Brown, “General consideration of tower antennas for broadcast use,” *Proc. I.R.E.*, vol. 23, pp. 311–356; April, 1935.

² L. S. Palmer and K. G. Gillar, “The distribution of uhf currents in long transmitting and receiving antennae,” *Proc. IEE*, vol. 13, p. 285; 1938.

³ G. Barzilai, “Experimental determination of current and charge along cylindrical antennas,” *Proc. I.R.E.*, vol. 37, pp. 825–829; July, 1949.

the normal component of the electric field. Accordingly, a small dipole protruding from the slotted inner conductor is used as a charge-indicating probe.

Fig. 2 is a block diagram of the measuring setup. For the transmitting antenna, connection AB is made. The signal source is a 300-Mc amplitude-modulated transmitter. The signal picked up by the probe is detected by

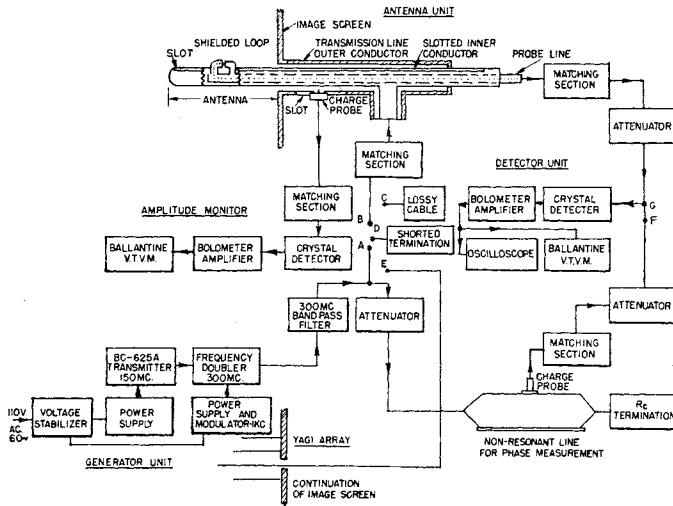


Fig. 2—Block diagram of measuring setup.

a 1N21B crystal. In the case of the receiving antenna a Yagi array located $7\frac{1}{2}$ wavelengths away is used to excite the unit of Fig. 1. To obtain the various loads for the receiving antenna, connection BD or BC is made. By measuring the standing-wave distributions in the line and referring the impedance to the antenna terminals, any desired loading may be obtained.

B. Phase Measurements

For a dissipationless transmission line terminated in its characteristic impedance, the current and voltage distributions at any point x along the line are given by the form

$$A = A_0 e^{-j\beta x},$$

so that the phase is retarded linearly with x while the amplitude remains constant. If the signal whose relative phase is to be determined is mixed with the reference signal from the probe on the Z_0 terminated line, the resultant signal is the vector sum of the two signals. By moving the sliding probe along the "flat" line to a point where the two signals are 180 degrees out of phase, a minimum indication is obtained on the meter which establishes a reference point for measuring the relative phase. A variation in phase of the probe signal is compensated by a measured displacement Δx of the sliding probe, so that the relative phase difference $\Delta\phi$ between any two points is given by

$$\Delta\phi = \frac{2\pi\Delta x}{\lambda_g},$$

where λ_g is the wavelength inside the nonresonant transmission line. In Fig. 2, phase measurement is effected by connection GF .

III. MEASURED RESULTS

A. Transmitting Antenna

In Figs. 3 and 4 are shown the measured distributions of current and charge, together with their phases both inside the transmission line and on the antenna, for antenna lengths $\beta_0 h = \pi/2$ ($h = \lambda/4$) and $\beta_0 h = \pi$ ($h = \lambda/2$),

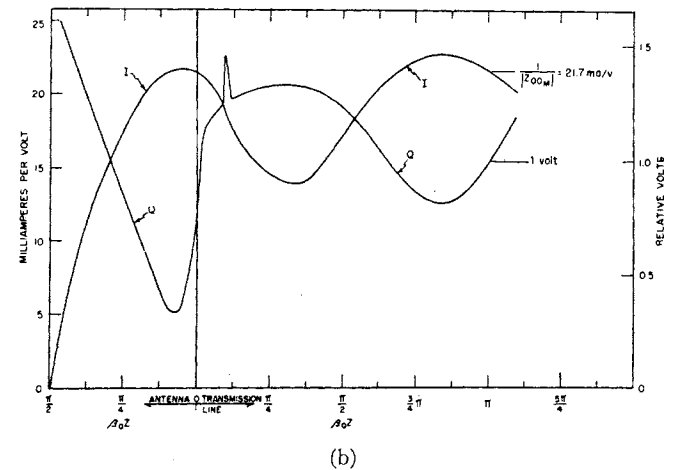
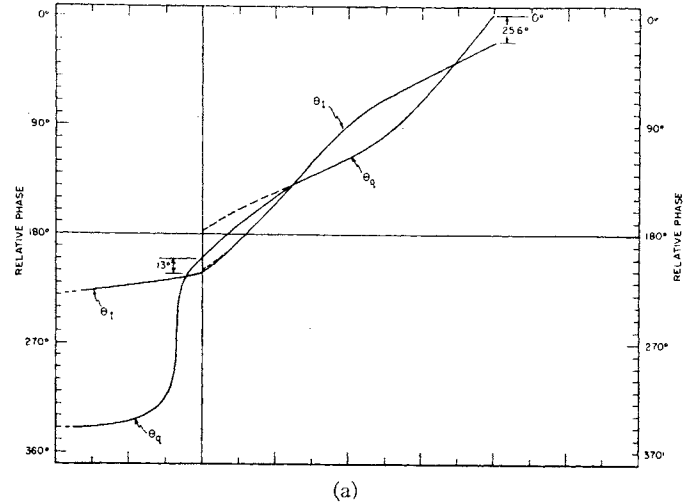


Fig. 3—Transmitting antenna current and charge distribution, $h = \lambda/4$.

$$(a) \beta_0 h = (\pi/2). \quad \Omega = 2 \log (2h/a) = 10.12$$

$$Z_{00} = 43 + j22.5 = 58.5 \angle 27.7^\circ$$

$$Z_{00M} = 41.9 + j20 = 46 \angle 25.6^\circ$$

$$\theta_q = \text{phase of charge distribution}$$

$$\theta_i = \text{phase of current distribution}$$

$$(b) \quad I = \text{current distribution}; \quad Q = \text{charge distribution.}$$

with $\beta_0 = 2\pi/\lambda$. Inside the transmission line the current and charge distributions behave in accordance with conventional transmission-line theory, with the current maximum occurring at the point of charge minimum. By taking the ratio of maximum to minimum of either current or charge distribution, and noting the position of the minimum with respect to the antenna terminals,

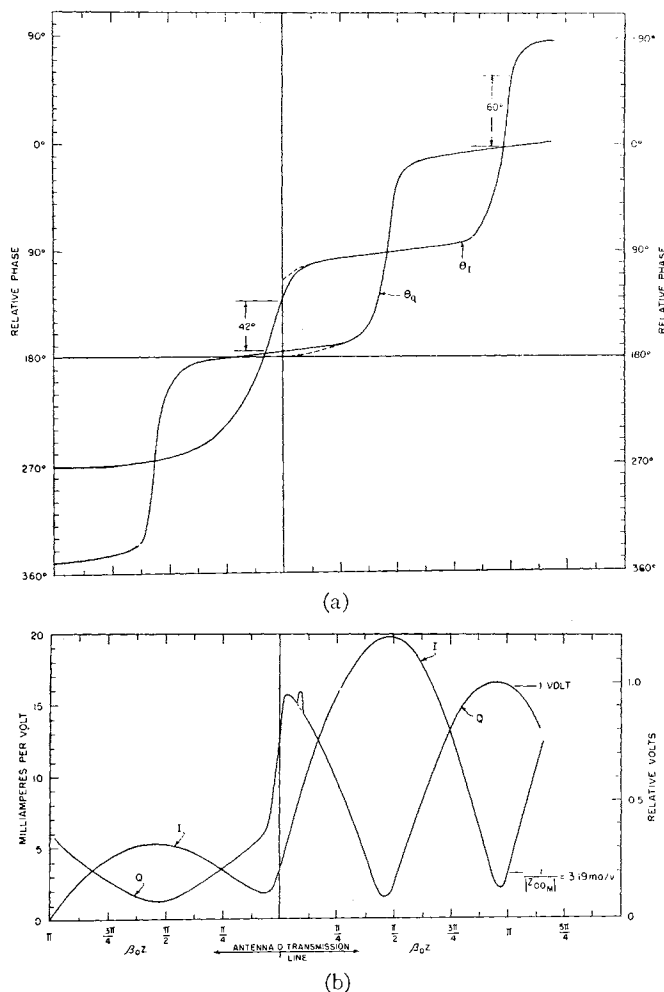


Fig. 4—Transmitting antenna current and charge distribution, $h = \lambda/2$.

(a) $\beta_0 h = \pi$, $\Omega = 11.5$

$$Z_{00} = 206 - j300 = 364 \angle -55.5^\circ$$

$$Z_{00M} = 157 - j272 = 314 \angle -60^\circ$$

θ_q = phase of charge distribution

θ_I = phase of current distribution

(b) I = current distribution; Q = charge distribution.

an impedance for the antenna load may be obtained. This is indicated as Z_{00M} in the figure. This measured value of impedance corresponds to the measured impedance exactly a half wavelength from the gap. The necessity for defining impedance in this manner may be seen from the wave picture inside the transmission line. At some distance back of the gap, only the dominant *TEM* mode is present. At the gap, however, higher-order modes are introduced to satisfy the boundary conditions at the discontinuity. That this is the case may be seen by the abrupt change in charge distribution and in the behavior of the phase distribution at the discontinuity. The sharp rise in charge distribution at a point $\beta_0 z = 0.314$ from the gap must not be confused with the gap discontinuity, however; it is to be attributed to the change in the dielectric constant due to the 1/8-inch polystyrene bead support for the inner conductor.

At a half wavelength from the gap, where the impedance is defined, only the dominant *TEM* mode is present, so that the charge distribution is proportional to the voltage distribution. Accordingly, if the voltage at this point is arbitrarily set at 1 volt, the admittance gives directly the current amplitude in milliamperes per volt. Further, at a point of current and charge maximum (or current and charge minimum), the current and charge are in phase. Thus, even though two independent measurements have been made for relative current and charge phase distribution, these two distributions can be directly correlated. Inside the transmission line, the phase distribution curves thus intersect at distances corresponding to a quarter wavelength. At any point, the phase difference between current and charge gives the phase angle of the impedance presented at that point by the antenna. For example, at a half wavelength from the gap, the respective phase differences of 26 degrees and 60 degrees for $\beta_0 h = \pi/2$ and π give the phase angle of the apparent impedance Z_{00M} . This differs slightly from the phase angle of the antenna impedance Z_{00} since the latter value has been corrected for the change in dielectric constant due to the polystyrene support. For $\beta_0 h = \pi/2$, the impedance of the antenna is close to that of the characteristic impedance of the transmission line ($Z_c = 60.6 \Omega$) so that the standing-wave ratio is not very high ($\text{SWR} = 1.65$) nor the phase variation very rapid. For $\beta_0 h = \pi$, the standing-wave ratio is high ($\text{SWR} = 10.6$) and the phase variation correspondingly rapid at the point of current minimum. For a line terminated in its characteristic impedance the phase varies linearly with distance; for a line terminated in an open or short circuit the phase varies abruptly as a step function.

At the gap terminals the deviation from normal transmission-line behavior may be seen from a comparison of the actual phase deviation with the normal phase deviation shown by the dotted line. The normal phase distribution is obtained from the distribution back of the gap where only the dominant *TEM* mode is present. The large mismatch between antenna and transmission line is noticeable for the $\beta_0 h = \pi$ case, where the maximum current amplitude inside the transmission line is about four times that of the maximum amplitude on the antenna. For $\beta_0 h = \pi/2$ the maximum amplitudes are nearly equal.

In Fig. 5 is shown the measured impedance, corrected for the bead support but not for the discontinuity at the gap terminals, compared with the theoretical values from the second-order King-Middleton solution for the cylindrical antenna.⁴ The theoretical model is based on the antenna center-driven by a slice generator of scalar-potential difference

$$V_0^e = \lim_{z \rightarrow 0} (\phi_{+z} - \phi_{-z}).$$

⁴ R. W. P. King and D. Middleton, "The cylindrical antenna, current and impedance," *Quart. Appl. Math.*, vol. III; January, 1946.

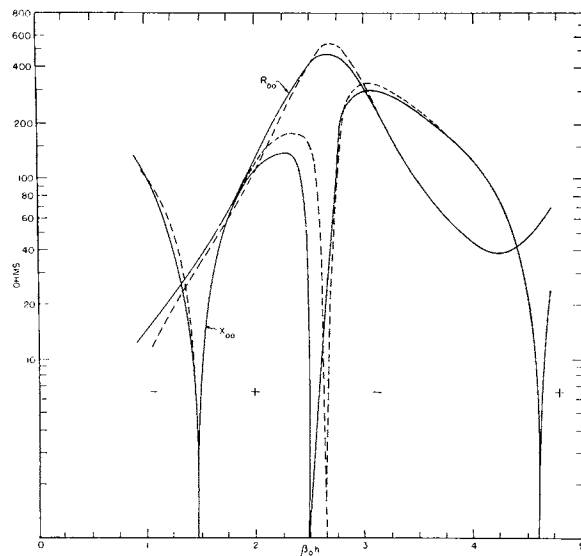


Fig. 5—Impedance curve for cylindrical dipole over ground screen; ——— measured; ——— theoretical (King-Middle second order).

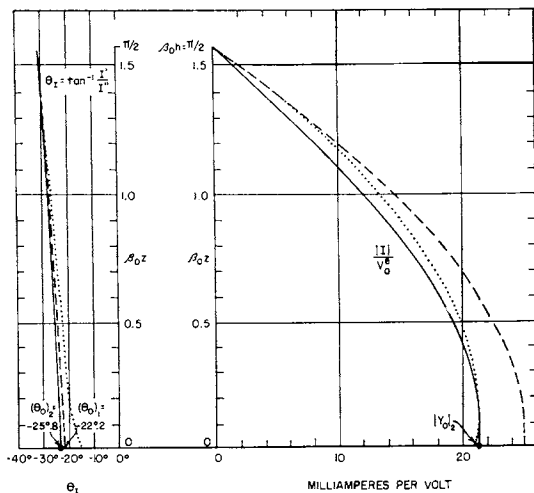


Fig. 6—Comparison of measured and theoretical current distribution, $h = \lambda/4$. ——— Second order (approximate); ——— first order $\Omega = 10$; experimental $\Omega = 10.12$.

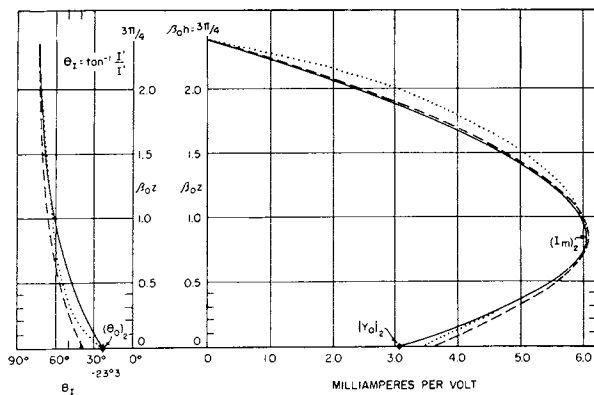


Fig. 7—Comparison of measured and theoretical current distribution, $h = 3/8\lambda$. ——— Second order (approximate); ——— first order $\Omega = 10$; experimental $\Omega = 10.76$.

In Figs. 6 through 9, the measured distributions for respective antenna lengths of $\beta_0 h = \pi/2$, $3\pi/4$ and $5\pi/4$ are plotted against the first-order and also the approximate second-order current distribution obtained from

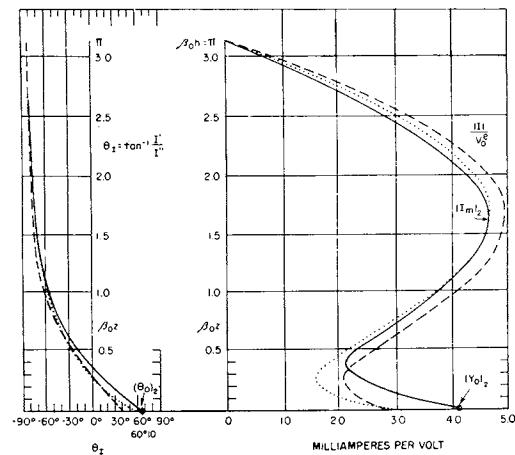


Fig. 8—Comparison of measured and theoretical current distribution, $h = \lambda/2$. ——— Second order (approximate); ——— first order $\Omega = 10$; experimental $\Omega = 11.5$.

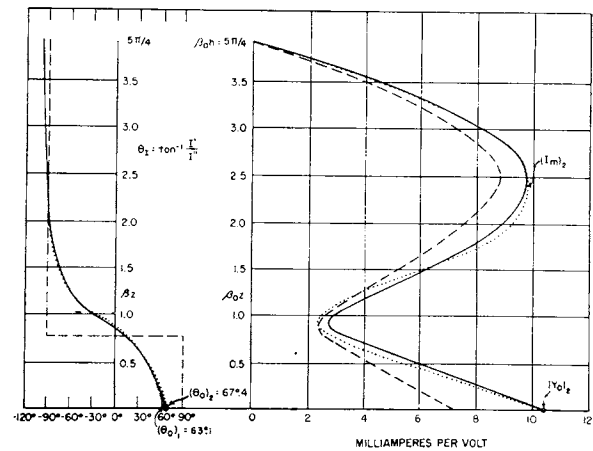


Fig. 9—Comparison of measured and theoretical current distribution, $h = 5/8\lambda$. ——— Second order (approximate); ——— first order $\Omega = 10$; experimental $\Omega = 11.94$.

the King-Middleton solution. Since there is no method for taking into account the discontinuity at the gap terminals, the measured amplitude curves have been normalized at the maximum of the theoretical curve, while the measured phase has been normalized at the end of the antenna. The phase variation on the antenna checks closely, except near the gap where the terminal-zone effect is present. Actually, in the absence of any terminal-zone effect, the current amplitude given in milliamperes per volt is identical with the absolute value of the antenna admittance, while the phase angle at the antenna terminals is the same as the phase angle of the admittance. Since the solution for the second-order distribution is excessively complicated, an approximate method based on the first-order distribution curves was used to obtain the theoretical curve. The input current of the first-order current distribution was corrected by the second-order impedances, while the maximum current on the antenna was found by assuming it to be proportional to the radiation resistance for the second-order theory. With these two points established, a curve based on the first-order distribution was

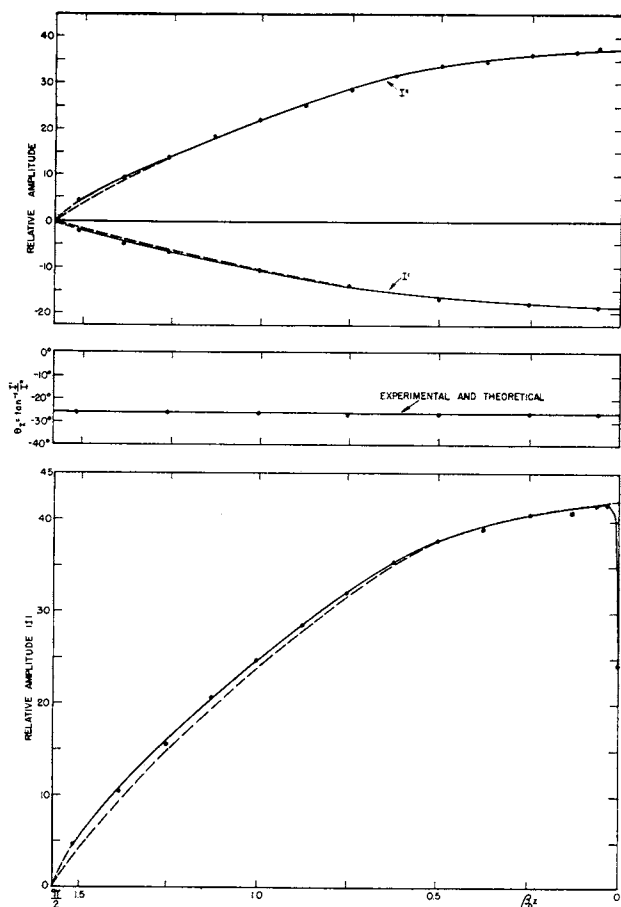


Fig. 10—Unloaded receiving antenna current distribution, $h=\lambda/4$. $\beta_0 h = (\pi/2)$. $\Omega=10.12$. — Measured; - - - King-Middleton ($\Omega=10$).

drawn in. The method, though in no sense rigorous, does nevertheless yield results which show the correct trend.

B. Receiving Antenna

Hitherto it has been customary in the treatment of the receiving antenna problem to assume the current along the antenna, whether loaded or unloaded, to be the same as a center-driven antenna of the same length. The assumption is based upon the indiscriminate use of the reciprocity theorem, the argument being that since the receiving and transmitting field patterns are identical, the current distributions must be the same. Von Korshenewsky⁵ first pointed out that when a plane wave is incident upon an unloaded antenna the current distribution is different from that of a transmitting antenna. By a fundamentally more rigorous analysis, Hallén⁶ and King and Harrison⁷ arrived at this same conclusion. It should be pointed out, however, that even though the current distributions may be different the impedances for transmission and reception are the same. This has been experimentally verified.⁸

⁵ N. Von Korshenewsky, "On the vibration of an oscillator in a radiation field," *Z. Tech. phys.*, vol. 10, p. 604; December 1929.

⁶ E. Hallén, "Theoretical investigation into the transmitting and receiving qualities of antennas," *Nova Acta Upsala*, vol. II, pp. 1-44; November, 1938.

⁷ C. W. Harrison, Jr., and R. King, "The receiving antenna," *Proc. I.R.E.*, vol. 32, pp. 18-35; January, 1944.

⁸ D. G. Wilson, "Impedance Measurements on a Receiving Antenna," Doctoral Thesis, Harvard University; December, 1947.

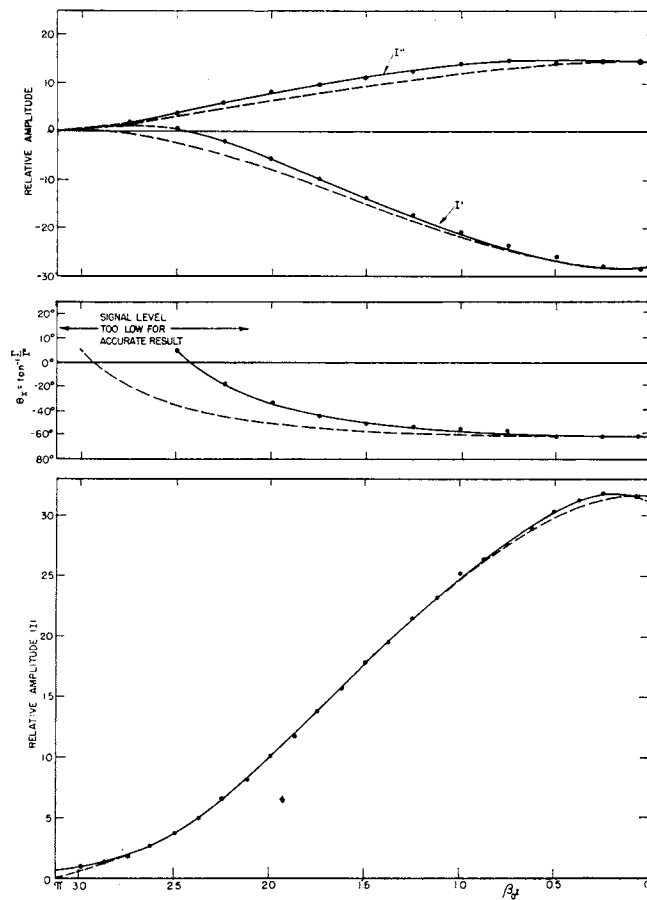


Fig. 11—Unloaded receiving antenna current distribution, $h=\lambda/2$. $\beta_0 h = \pi$. $\Omega=11.5$. — Measured; - - - King-Middleton ($\Omega=10$).

The general solution of the integral equation obtained when a plane wave is incident upon an unloaded antenna is a superposition of two solutions

$$I(z) = I(-z)$$

and

$$I(z) = -I(-z).$$

The distribution of current obtained by using the symmetrical solution is unidirectional through the center of the antenna so that if a load is placed at the center a potential difference can be established across it. For the antisymmetrical solution, on the other hand, the current reduces to zero at the center and the currents in both halves flow simultaneously toward or away from the center so that no potential difference is established at the load. Since the total current is given by the superposition of these two solutions, the symmetrical solution describes only the total current at the center. For the special case in which the receiving and transmitting antennas are parallel—i.e., the incident plane wave is parallel to the receiving antenna—the antisymmetrical current vanishes and the total current is given only by the symmetrical component. Moreover, in the measurements to be described, the antennas are placed over a ground screen and hence from the principle of images it follows that only the symmetrical component can exist.

Unloaded Receiving Antenna: In Figs. 10 and 11 are shown the measured distributions for the unloaded receiving antenna ($Z_L=0$) for $\beta_0 h = \pi/2$ and π , compared with the theoretical curves of King. The correlation here is seen to be excellent. For this type of receiving antenna there is no gap terminal effect, since the antenna is short-circuited to the ground screen. The sharp drop in amplitude at $\beta_0 z = 0$ is due to the fact that the probe moves into the ground screen through a small slot. The currents I' and I'' are the components in phase and in phase quadrature with the incident E field obtained from the amplitude and phase data. For $\beta_0 h = \pi$, the distribution is seen to be completely different from that of the transmitting case shown in Fig. 4, with a current maximum occurring at the ground screen.

The Loaded Receiving Antenna: The effect of the unloaded receiving antenna considered above is merely to scatter the incident electromagnetic field after dissipating a small fraction of the total incident energy in ohmic losses due to finite conductivity of the antenna. In normal operation the receiving antenna is connected to a load impedance so that part of the incident energy will be utilized in the load, a small part dissipated in ohmic loss, and the rest re-radiated. With the aid of the compensation theorem it may be shown that the current distribution function at any point z along the receiving antenna, for any arbitrary load, can be decomposed into two functions $f_E(z)$ and $f_V(z)$, where $f_E(z)$ is the distribution function for the unloaded receiving antenna and $f_V(z)$ is the distribution function for the transmitting antenna. The amplitude and phase of the transmitting distribution $f_V(z)$ are functions of the terminating load impedance so that the actual distribution along the receiving antenna may be varied by varying the load impedance.

In Fig. 12 is shown the equivalent circuit for the receiving antenna at the terminals. The expression for I_0 , the current at the antenna terminals, is given by

$$I_0 = \frac{V}{Z_{00} + Z_L}.$$

From this equivalent-circuit concept three important results follow:

1. The condition for maximum I_0 , when Z_L is variable, is the conventional resonance condition

$$Z_L = -jX_{00}.$$

For this case a maximum broadside re-radiation occurs.

2. When the antenna is terminated in its conjugate impedance,

$$Z_L = Z_{00}^*.$$

The power dissipated in the load is then equal to the power re-radiated, and maximum power transfer to the load occurs.

3. When the receiving antenna is terminated in an

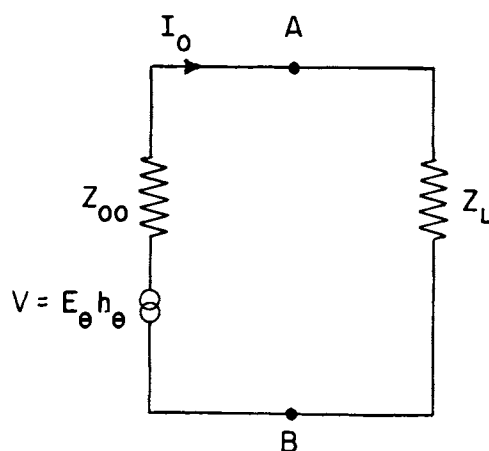


Fig. 12—Equivalent circuit of receiving antenna.
 Z_L = load impedance
 $Z_{00} = R_{00} + jX_{00}$ (self-impedance of antenna)
 E_θ = incident electric field
 h_e = effective height of antenna.

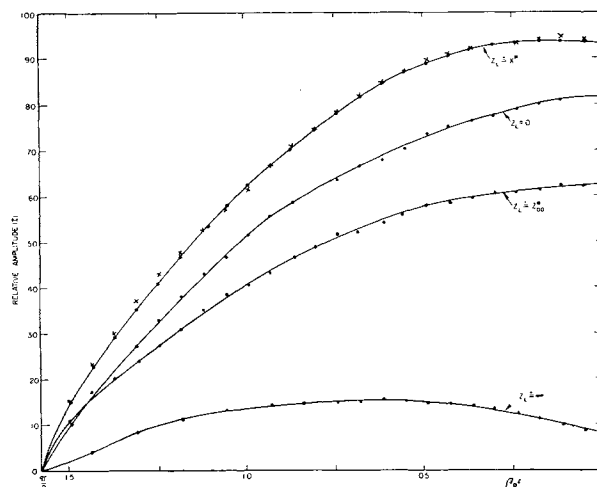


Fig. 13—Current distribution on receiving antenna for various loads. $\beta_0 h = (\pi/2)$. $\Omega = 12$. $E_{\text{incident}} = \text{constant}$. X = measured transmitting antenna current distribution.

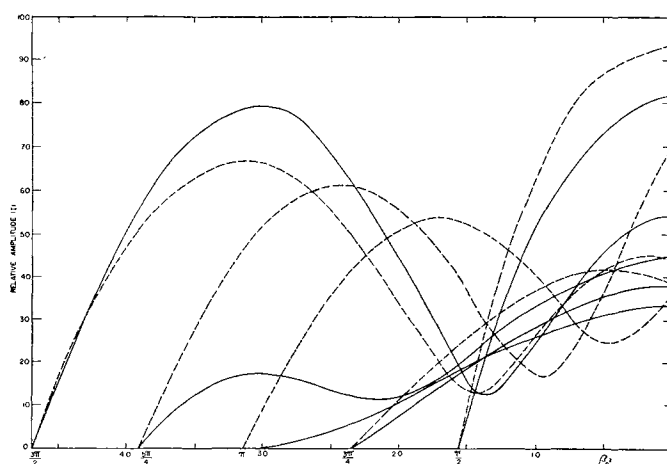


Fig. 14—Current distribution on receiving antenna. — $Z_L = 0$
 --- $Z_L = X_{00}^*$ with $E_{\text{incident}} = \text{constant}$.

open circuit

$$Z_L = \infty,$$

the current at the antenna terminals is zero.

To illustrate the three conditions, Fig. 13 shows the measured distribution for a quarter-wavelength antenna terminated by these three loads for the same incident E field. The top curve, showing the antenna terminated in its conjugate reactance, is compared with the distribution for the transmitting antenna given in Fig. 3. As predicted by theory, the transmitting-antenna distribution $f_V(z)$ predominates over the unloaded receiving-antenna distribution $f_E(z)$ shown below it for $Z_L = 0$, and agrees very closely with the measured transmitting antenna current distribution. The open-circuited case $Z_L = \infty$ is only approximate because of the gap effect. This distribution was obtained by leaving the probe at the gap terminals and tuning a short-circuited section of transmission line for minimum current at the gap terminals. These results illustrate the wide range of distributions possible as functions of the load.

Finally, in Fig. 14 are the measured distributions for antennas of the respective lengths $\beta_0 h = \pi/2$, $3\pi/4$, π , $5\pi/4$, and $3\pi/2$ for the unloaded and conjugate reactance terminations for the same incident E field. For $\beta_0 h = \pi/2$ and $\beta_0 h = 3\pi/2$ the current amplitudes are seen to be largest, while for $\beta_0 h = \pi$ the amplitude is very small. This indicates that the broadside scattering

should be greatest in the region of $\beta_0 h = \pi/2$ and $3\pi/2$, which is in agreement with the curves obtained by Van Vleck, Bloch, and Hamermesh.⁹

The effect upon the measured result of the slotted antenna, finite ground screen, probe loading, and the deviation of the incident E field from a plane wave front is discussed elsewhere.¹⁰

IV. CONCLUSION

A method has been presented for measuring the distribution of current and charge which gives not only the amplitude but also the phase on the antenna and on the transmission line. The method described is applicable to many other antennas which may be driven over a ground screen. At a later date a paper will be presented showing the results of current-distribution measurements on folded dipoles and closely coupled antennas.

ACKNOWLEDGMENT

The author gratefully acknowledges the guidance and encouragement of R. W. P. King of the Cruft Laboratory during the course of this investigation.

⁹ J. H. Van Vleck, F. Bloch, and M. Hamermesh, "Theory of radar reflections from wires or thin metallic strips," *Jour. Appl. Phys.*, vol. 18, pp. 274-294; March, 1947.

¹⁰ T. Morita, "The Measurement of Current and Charge Distributions on Cylindrical Antennas," Technical Report No. 66, Cruft Laboratory, Harvard University, February 1, 1949.



Correction and Clarification of

"Automatic Volume Control as a Feedback Problem"*

Bernard M. Oliver, author of the above-mentioned paper, has called to the attention of the editors an error in Fig. 7, and also noted that the paragraph following equation (19) is misleading. This equation is correct as given, but in discussing its significance he feels that it would be preferable to state the following:

"Hence the loop gain in an 'undelayed' system may be expressed solely in terms of the rf amplifier control characteristic and the operating control voltage. Two systems differing only in the amount of automatic volume control (avc) path amplification, $\mu_1(0)\beta(0)$, will have the same loop gain and the same rf amplification M when the control voltages V are equal. However this equality of control voltages will occur with different values of $\bar{e}_1$ and $\bar{e}_2$ in the two systems. Suppose, for example, that in system A , $\mu_1(0)\beta(0) = 1$, while in sys-

tem B , $\mu_1(0)\beta(0) = 10$. Then the same control voltage and loop gain will exist in both systems when $\bar{e}_1$ and $\bar{e}_2$ are $1/10$ as great in system B as in system A . The higher $\mu_1(0)\beta(0)$ is made, the smaller the value of $\bar{e}_1$ required to produce a given loop gain."

From this it is apparent that the dotted curve (for the undelayed system) in Fig. 7 should be obtained by shifting the solid curve down one decade *and to the left one decade*. This latter shift was omitted in the published drawing. Obviously, all the curves should coincide for values of $\bar{e}_1$ so small that no avc action is produced. Since the curves are convex upward, the system with $\mu_1(0)\beta(0) = 10$ has more loop gain at any given value of $\bar{e}_1$ or $\bar{e}_2$ than the system with $\mu_1(0)\beta(0) = 1$, though not a great deal more.

The author wishes to thank Arno M. King of the Naval Research Laboratory who first pointed out these errors, and to apologize for any difficulties they may have caused.

* B. M. Oliver, "Automatic volume control as a feedback problem," *Proc. I.R.E.*, vol. 36, pp. 466-474; April, 1948.